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MODELING TEMPERATURE FIELDS IN GLASS ARTICLES IN ANNEALING

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Methods for the calculation of temperature fields in flat and cylindrical glass articles are considered. The advantages of numerical methods compared to analytical methods for the calculation of temperature fields in complex shapes of articles and heat-exchange processes are demonstrated.

The most essential aspect in selecting an annealing regime is the development of mathematical models for calculation of temperature fields in glass articles of various shapes.

Glass is a semitransparent material, in which energy is transferred by two vehicles: conduction and radiation. Energy transfer by conduction takes place due to energy exchange between neighbor atoms, whereas transfer by radiation is due to emission and absorption of radiant energy by atoms. Moreover, there is convective heat exchange between the glass surface and the ambient medium and heat exchange by radiation between glass layers and enclosing surfaces [1].

A rolled glass sheet can be considered as an infinite plate, whose thickness is significantly less than its width and length; therefore, in constructing a mathematical model for calculation of the temperature field in a glass band, it is assumed that the conductive heat transfer process is one-dimensional (across the glass thickness).

In the case of nonstationary heat flow (a plate being cooled or heated), conductive heat transfer is described by the differential equation of thermal conductivity:

$$\frac{\partial T(x, \tau)}{\partial \tau} = a\frac{\partial^2 T(x, \tau)}{\partial x^2},$$

where $T(x, \tau)$ is the temperature function in the coordinate x across the plate thickness at the time moment τ; a is the temperature conductivity of glass, m^2/sec.

The convective heat exchange between the plate end the ambient medium is described by the Newton law, according to which the heat flow density $q_c(\tau)$ is proportional to the difference between the temperatures of the body surface $T_s(\tau)$ and ambient medium $T_m(\tau)$ [2]:

$$q_c(\tau) = \alpha[T_s(\tau) - T_m(\tau)],$$

where α is the convective heat exchange coefficient, $W/(m^2 \cdot K)$.

For the given law of ambient temperature variation in conductive-convective heat exchange, the solution of the differential equation of thermal conductivity can be obtained by the Laplace integral transform method or by analytical separation of variables, which makes it possible to calculate the temperature distribution across the glass band thickness at any given moment directly from the formulas. The drawbacks of solutions described in the literature [2 – 4] is the fact that they do not take into account the asymmetry of heat exchange conditions for the lower and the upper glass surfaces and heterogeneity of initial temperature distribution in annealing of glass articles.

The fact is that after coming from beneath the shaping rolls, the glass band arrives onto a cooling plate or on support rolls. Its cooling before the annealing furnace occurs in asymmetric conditions, since the glass band is cooled from beneath by the cooling plate surface (or by rotating support rolls) and its upper surface in cooled by the ambient medium [5]. There is also asymmetry of thermal conditions inside the convective annealing furnace due to the vertical air flow. At the point of glass transition from the bulb zone chilled by the shaping rolls to the band zone (on the projection of the shaping rolls axis) the temperature in the middle of the plate significantly exceeds the temperatures on its surfaces [5], as a consequence of which the initial temperature distribution in

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the band is close to a parabolic one and significantly differs from a uniform distribution.

To overcome these drawbacks, it is necessary to consider the most general case of asymmetric convective heat exchange with a linearly changing medium temperature and a parabolic initial temperature distribution [6]. The formulas are derived for the case of constant thermophysical properties and do not take into account heat exchange by radiation; therefore, they can be applied only to calculate temperature distributions in a glass band annealed in a convective furnace.

It should be noted that a significant contribution to external heat exchange of glass in molding and annealing in a convection-radiation furnace is made by the radiation component of the heat flow $q_r(\tau)$, which is proportional to the difference of the fourth powers of the absolute temperatures of the glass surface $T_s(\tau)$ and the heat-absorbing body (annealing furnace heaters or enclosing surfaces) $T_h(\tau)$ [1]:

$$q_r(\tau) = \varepsilon_g \sigma [T_s^4(\tau) - T_h^4(\tau)],$$

where ε_g is the degree of blackness of the glass surface; $\sigma = 5.67 \times 10^{-8}$ W/(m$^2 \cdot$ K^4) is the Stefan – Boltzmann constant.

In annealing glass articles, the initial temperature distribution is nonunifom. Furthermore, radiation annealing furnaces also have asymmetry of thermal conditions due to different conditions of radiation heat exchange of the glass band with bottom or roof heaters. Therefore, it is necessary to consider the case of an asymmetric convection-radiation heat exchange of the band, which in the theoretical aspect means transition to a problem with nonlinear boundary conditions.

The calculation method proposed in [7] using effective thermal conductivity values, which take into account both conductive and radiation heat transfer, is approximate; moreover, it does not take into account the physical meaning of the process of heat transfer by radiation. The classical methods (Laplace, separation of variables) are not suitable for this particular problem either; therefore, the study in [6] considers the method of finite integral transforms, which is the most efficient for the specified purpose.

An account of the radiation component leads to nonlinear boundary conditions for heat flows, as a consequence of which a solution for the thermal conductivity differential equation using the Koshlyakov – Grinberg finite integral transform method will be a system of two integral Volterra equations of the second kind with respect to the plate surface temperatures. To find the surface temperatures from the system of integral equations, an iteration procedure using an analog of the Newton – Raphson method is applied. The obtained surface temperatures make it possible to find the temperature distribution inside the plate at any time moment [6].

The considered model for calculating the temperature field in a plate in asymmetric convection-radiation heat exchange is "nontransparent," since it does not take into account radiation transfer inside the glass. Models for calculating temperature fields in glass taking into account radiation heat exchange inside it have so far been developed only for sheet material with symmetric thermal fields. The best known is the numerical calculation method developed in [8, 9] describing a symmetrical case of radiation heat exchange (the temperatures of enclosing surfaces beneath and above the glass band are assumed to be equal).

Let us carry out a comparative analysis of advantages and disadvantages of analytical and numerical methods for calculation of temperature fields.

The main advantages of analytical methods is the fact that the solution is obtained in a form allowing for calculation of the temperature distribution in the plate at any point for any time moment. In using numerical methods, temperature distribution for any moment can be obtained knowing this distribution at the preceding time moment, i.e., by the iteration method.

At the same time, any complication of the initial (from a uniform initial temperature distribution changing to a parabolic distribution, from the parabolic one to a polynomial of a higher order) and boundary (from a constant to a linearly changing medium temperature) conditions or account of the temperature dependences of glass properties (thermal conductivity, temperature conductivity) make it significantly more difficult to solve the differential equation of thermal conductivity using analytical methods. Transition to nonlinear boundary conditions, which take into account radiation heat exchange, produces a situation in which the solution constitutes a system of integral equations related to the plate surface temperatures, which are found by an iteration procedure, thus canceling the chief advantages of the derived analytical formulas. Transition to a "semitransparent' model that takes into account radiation heat transfer inside the glass makes it impossible to obtain a solution by analytical methods.

The numerical scheme for solving the thermal conductivity differential equation by the finite difference method is free from the above drawbacks. Furthermore, the author's experience shows that numerical methods for calculation of temperature fields in glass are significantly faster. Therefore, in modeling a temperature field in a glass band for the most general case of asymmetrical convection-radiation heat exchange taking into account the semitransparency of glass, one should use a numerical calculation method [6].

Modeling of temperature fields in cylindrical glass articles (a glass, a bottle) by analytical methods involves substantial difficulties even without taking into account radiation heat transfer in glass. This is due to the more complicated form of the differential equation of thermal conductivity in cylindrical coordinates:

$$\frac{\partial T(r,h,\tau)}{\partial \tau} = a\left[\frac{\partial^2 T(r,h,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,h,\tau)}{\partial r} + \frac{\partial^2 T(r,h,\tau)}{\partial h^2}\right],$$

where $T(r, h, \tau)$ is the temperature at a point with coordinates along the radius r and height h at a time moment τ, as well as

due to the circumstance that a cylindrical glass article in fact consists of several cylindrical bodies.

In particular, a bottle consists of the lateral and the neck parts (hollow cylinders), a bottom part (cylinder), and a transitional part (a hollow cone). This creates complex boundary conditions for the surface points of the article and for the points of transition from one cylindrical body making part of the article to another.

To calculate temperature fields in cylindrically shaped glass articles, the authors in [1, 7] suggest applying the model of the temperature field in a glass band using what is known as effective thickness of an article instead of the glass band thickness. The calculation in this case is significantly less accurate, especially due to complex heat exchange conditions of articles placed in many rows on a transporter grid of the annealing furnace.

We propose using a numerical method for calculating temperature fields in cylindrical glass articles [6], which makes it possible to overcome the modeling difficulties related to the complicated shape of the articles. At the same time, one should take into account radiation heat exchange between cylindrical articles on a technological line.

Thus, numerical calculation methods are preferable to analytical ones for complex shapes of glass articles and for complicated heat exchange conditions, due to both their simplicity and fastness.

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